Ranking Alternatives by an Extension to Fuzzy VIKOR

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Abstract: An extension to fuzzy VIKOR is proposed, where membership functions of fuzzy weighted weightings can be derived. The proposed extension can resolve the limitation of using approximation to represent multiplication of two positive triangular fuzzy numbers in existing methods. The total integral value is used to rank fuzzy numbers and formulas of ranking procedure are presented to help complete the proposed model. Finally, a numerical example is used to demonstrate the feasibility of the proposed extension.

Keywords: Fuzzy VIKOR, membership functions, total integral value, ranking

1. Introduction

The method of VIKOR was developed for multi-criteria optimization to obtain the best feasible solution in Multiple Criteria Decision-Making (MCDM) problems [1], indicating that VIKOR is an MCDM method. In many situations, alternatives versus qualitative criteria and criteria weights are usually assessed in linguistic values represented by fuzzy numbers, leading to study of fuzzy VIKOR [2, 3].

Numerous methods and applications have been investigated by fuzzy VIKOR [4, 5] and fuzzy VIKOR was also extended to intuitionistic environment to resolve many problems, such as robot selection [6]. Despite the merits, most of the existing fuzzy VIKOR methods have the limitation of using approximation for the multiplication of fuzzy ratings and fuzzy weights in establishing the model. To overcome the above limitation, this paper proposes an extension of fuzzy VIKOR, in which the alternative ratings under qualitative criteria and the weights of criteria are evaluated via linguistic values that are quantified using positive triangular fuzzy numbers; additionally, positive triangular fuzzy numbers are also used to evaluate the alternative ratings under quantitative criteria. In the proposed fuzzy VIKOR extension, formulas for the membership functions of two positive triangular fuzzy numbers can be developed for better implementation of the decision-making process. In the above formula derivation, the smallest value which is usually a negative value, from the result of subtractions of two triangular fuzzy numbers is added to all elements in the decision matrix to avoid any negative or partially negative triangular fuzzy numbers. This is due to the reason that exact formula cannot be developed for the multiplication of two negative or partially negative triangular fuzzy numbers. In addition, the method of total integral value [7] is used to conduct the defuzzification for ranking fuzzy numbers to complete the model, and formulas for the defuzzification can be derived. Finally, a numerical example will be used to demonstrate the feasibility of the proposed method.

The rest of this paper is organized as follow. Section 2 presents the literature review. Section 3 introduces the proposed method, and a numerical example is conducted in Section 4 to demonstrate the feasibility of the proposed method. Finally, Section 5 presents the conclusion.

2. Literature Review

The name of VIKOR [8] in Serbian is "VIseKriterijumska Optimizacija I Kompromisno Resenje", which means "Multicriteria Optimization and Compromise Solution" in English. The first real application of VIKOR was presented in 1998 [9]. VIKOR can be regarded as a useful MCDM technique [1, 10], which can be used to resolve complex decision-making problems. However, when using VIKOR to solve problems in uncertain environment, decision-makers need to deal with the ratings of alternatives versus qualitative criteria and criteria weights using linguistic values. The ratings of alternatives versus qualitative criteria and criteria weights are usually assessed in linguistic values which can be represented by fuzzy numbers, leading to the study of fuzzy VIKOR [2, 3]. Moreover, the ratings of alternatives under quantitative criteria can also be fuzzy and these ratings can also be assessed through triangular fuzzy numbers. Methods and applications of fuzzy VIKOR have been extensively investigated, such as identification of cloud computing risks [5] and evaluation of risks in foodgrains supply chain [11]. Some different applications and models of fuzzy VIKOR can be seen in [12].

The method of VIKOR was extended in intuitionistic fuzzy environment [6], in which criteria weights and ratings of alternatives were represented by triangular intuitionistic fuzzy sets. Many models and applications using intuitionistic fuzzy VIKOR have been investigated. Recent works include using intuitionistic fuzzy VIKOR and deferred acceptance algorithm to develop a matching mechanism for public cloud manufacturing platforms [13], selecting logistics center location via an extended VIKOR method based on interval-valued intuitionistic fuzzy numbers [14], sustainable supplier selection in organ transplantation networks for healthcare devices by an interval-valued intuitionistic fuzzy model based on extended VIKOR and MARCOS [15], a novel VIKOR approach in an intuitionistic fuzzy linguistic environment for the selection of renewable energy sources [16], and R&D project selection in defense industry investment decisions by an interval-valued intuitionistic fuzzy VIKOR approach [17] etc.

From the above literature, deriving formulas of the membership functions of the multiplication of fuzzy ratings and fuzzy weights has not been found yet. To fill this gap, this paper proposes ranking alternatives by an extension to fuzzy VIKOR. Fuzzy number ranking is an important step in fuzzy VIKOR. Numerous ranking methods have been investigated. Some various ranking methods and their applications can be seen in [18– 20]. Herein, the total integral value [7] is used not only because it is independent of the type of membership function used and the normality of the function, but also because its calculation is relatively simple.

3. Model Establishment

Concept of fuzzy set can be seen in [21]. Definition of fuzzy number can be seen in [22]. The α-cuts of a fuzzy number and arithmetic operations of fuzzy numbers can be seen in [23]. Linguistic variables and values can be seen in [24]. Assume that there are *k* decision-makers (i.e. D_t, *t*=1~*k*) to selection *m* alternatives (i.e. A_i, *i*=1~*m*) under *n* criteria (C_j, *j*=1~*n*). The criteria are categorized into qualitative and quantitative ones; where qualitative criteria are C_j, *j* = 1~*g*; while quantitative criteria are further classified to benefit criteria ($j \in B$), C_j, *j* =*g*+1~ *h*, and cost criteria ($j \in C$), C_j, *j* = *h*+1~*n*. Suppose ratings of alternatives versus qualitative criteria and criteria weights are assessed in linguistic values represented by positive triangular fuzzy numbers. Based on [4], an extension to fuzzy VIKOR is developed as the following steps.

Step 1. Average ratings of alternatives versus qualitative criteria

Let $x_{ijt} = \left(a_{ijt}, b_{ijt}, c_{ijt}\right)$, $x_{ijt} \in R^+$, $i = 1\sim m$, $j = 1\sim n$, $t = 1\sim k$. x_{ijt} is the rating assigned to alternative A_i by decision-maker D_t for criterion C_j. Each aggregated rating can be denoted as

$$x_{ij} = \left(a_{ij}, b_{ij}, c_{ij}\right) \text{ where } a_{ij} = \sum_{t=1}^{k} \frac{a_{ijt}}{k}, b_{ij} = \sum_{t=1}^{k} \frac{b_{ijt}}{k}, c_{ij} = \sum_{t=1}^{k} \frac{c_{ijt}}{k} \tag{1}$$

Step 2. Normalization of values under quantitative criteria

The values of quantitative criteria may be either crisp or fuzzy with different units, and those values must be normalized into a comparable scale for calculation rationale. In this paper, triangular fuzzy numbers are used. The approach from Ref. [25] is applied to complete the normalization. This approach preserves by property where the ranges of normalized triangular fuzzy numbers belong to [0,1]. Suppose $r_{ij} = (o_{ij}, p_{ij}, q_{ij})$ is the value of alternative A_i versus criteria C_j, $j=g+1 \sim n$. Values of normalization of r_{ij} are shown in Eqs. (2) and (3) as follows.

$$x_{ij} = \left(\frac{o_{ij} - \min_i o_j}{\max_i q_{ij} - \min_i o_j}, \frac{p_{ij} - \min_i o_j}{\max_i q_{ij} - \min_i o_j}, \frac{q_{ij} - \min_i o_j}{\max_i q_{ij} - \min_i o_j} \right), j \in B \tag{2}$$

$$x_{ij} = \left(\frac{\max_i q_{ij} - q_{ij}}{\max_i q_{ij} - \min_i o_j}, \frac{\max_i q_{ij} - p_{ij}}{\max_i q_{ij} - \min_i o_j}, \frac{\max_i q_{ij} - o_{ij}}{\max_i q_{ij} - \min_i o_j} \right), j \in C \tag{3}$$

Step 3. Determine fuzzy best value and fuzzy worst value

The fuzzy best value $f_j^* = (a_j^*, b_j^*, c_j^*)$ and fuzzy worst value $f_j^\circ = (a_j^\circ, b_j^\circ, c_j^\circ)$ can be determined by total integral value method [7]. $f_j^* = \max_i x_{ij}$ and $f_j^\circ = \min_i x_{ij}$ if C_j is a benefit criterion; $f_j^* = \max_i x_{ij}$ and $f_j^\circ = \min_i x_{ij}$ if C_j is a cost criterion.

Consider a fuzzy number A= [a, b, c] ∈R. The left and right inverse functions of A are denoted as $g_A^L(y)$ and $g_A^R(y)$, respectively. The left integral value is denoted as $I_L(A) = \int_0^1 g_A^L(y)dy$ and the right integral value is denoted as $I_R(A) = \int_0^1 g_A^R(y)dy$ The total integral value is defined as equation (4).

$$I_T^\lambda(A) = \frac{1}{2}(I_R(A) + I_L(A)), \text{ where } \lambda = 0.5. \tag{4}$$

For $A = (a,b,c)$, $g_A^L(y) = a + (b-a)y$, $g_A^R(y) = c + (b-c)y$, $I_L(A) = \frac{a+b}{2}$ and $I_R(A) = \frac{b+c}{2}$ can be obtained. The total integral value is obtained as equation (5). Thus, f_j^* and f_j° can be obtained by (5).

$$I_T^{0.5}(A) = \frac{1}{4}\left(a + 2b + c\right) \tag{5}$$

Step 4. Compute the normalized difference

The fuzzy difference d_{ij} is a triangular fuzzy computed as $d_{ij} = (f_j^* - x_{ij})/(c_j^* - a_j^\circ)$ where $c_j^* - a_j^\circ$ is a crisp value. To avoid negative values, the following equation is used.

$$d_{ij} = \frac{f_j^* - x_{ij}}{c_j^* - a_j^\circ} - \delta, \quad \delta = \min_{ij} \frac{f_j^* - x_{ij}}{c_j^* - a_j^\circ} \tag{6}$$

Step 5. Determine importance weights of criteria

Let $w_{jt}=\left(d_{jt},e_{jt},f_{jt}\right)$ be the weighted rating assigned by decision-maker D_t for criterion C_j. Each aggregated fuzzy weight can be denoted as

$$w_j=(d_j,e_j,f_j) \text{ where } d_j=\sum_{t=1}^{k}\frac{d_{jt}}{k},\ e_j=\sum_{t=1}^{k}\frac{e_{jt}}{k},\ f_j=\sum_{t=1}^{k}\frac{f_{jt}}{k} \tag{7}$$

Step 6. Compute the separation measures

$R_i=\left(R_i^a,R_i^b,R_i^c\right)$ as $R_i=\max_j\left(d_{ij}\otimes w_j\right)$ and $S_i=\left(S_i^a,S_i^b,S_i^c\right)$ as $S_i=\sum_{j=1}^{n}\left(d_{ij}\otimes w_j\right)$. First, the membership functions of $R_{ij}=d_{ij}\otimes w_j$ can be developed as follows. Let $d_{ij}=\left(a_{ij},b_{ij},c_{ij}\right)$ and $w_j=(d_j,e_j,f_j)$, i = 1~m, j =1~n, the membership functions of each weighted rating, $R_{ij}=d_{ij}\otimes w_{ij}$, can be derived based on arithmetic operations of fuzzy numbers [23] as $R_{ij}=$ $\left[(b_{ij}-a_{ij})(e_j-d_j)\alpha^2+\left(a_{ij}(e_j-d_j)+d_j(b_{ij}-a_{ij})\right)\alpha+a_{ij}d_j,\ (b_{ij}-c_{ij})(e_j-f_j)\alpha^2+\left(c_{ij}(e_j-f_j)+f_j(b_{ij}-c_{ij})\right)\alpha+c_{ij}f_j\right]$.

Suppose $I_{ij1}=\left(b_{ij}-a_{ij}\right).\left(e_j-d_j\right)$, $J_{ij1}=a_{ij}.\left(e_j-d_j\right)+d_j.\left(b_{ij}-a_{ij}\right)$, $I_{ij2}=\left(b_{ij}-c_{ij}\right).\left(e_j-f_j\right)$, $J_{ij2}=c_{ij}.\left(e_j-f_j\right)+f_j.\left(b_{ij}-c_{ij}\right)$, $Y_{ij}=b_{ij}.e_j$, $Z_{ij}=c_{ij}.f_j$. The membership functions of R_i are produced as:

$$f_{R_{ij}}^{L}(x)=\frac{-J_{ij1}+\left[J_{ij1}^2-4I_{ij1}\left(Q_{ij}-x\right)\right]^{1/2}}{2I_{ij1}},\ Q_{ij}\le x\le Y_{ij} \tag{8}$$

$$f_{R_{ij}}^{R}(x)=\frac{-J_{ij2}-\left[J_{ij2}^2-4I_{ij2}\left(Z_{ij}-x\right)\right]^{1/2}}{2I_{ij2}},\ Y_{ij}\le x\le Z_{ij}. \tag{9}$$

$R_i=\left(R_i^a,R_i^b,R_i^c\right)=\left[Q_{ij},Y_{ij},Z_{ij}\right]$. According to [7], the left inverse function of can obtained as $g_{R_{ij}}^{L}(y)=I_{ij1}y^2+J_{ij1}y+Q_{ij}$ and the right inverse function of can obtained as $g_{R_{ij}}^{R}(y)=I_{ij2}y^2+J_{ij2}y+Z_{ij}$. The left integral value can be produced as $I_L(R_{ij})=\frac{1}{3}I_{ij1}+\frac{1}{2}J_{ij1}+Q_{ij}$ and the right integral value can be produced as $I_R(R_{ij})=\frac{1}{3}I_{ij2}+\frac{1}{2}J_{ij2}+Z_{ij}$. Finally, the total integral value can be derived as equation (10). The defuzzification value of $R_i=\left(R_i^a,R_i^b,R_i^c\right)$, denoted as $\bar{R}_i^{\circ}$, can then be obtained by $\max_j I_T^{0.5}(R_{ij})$.

$$I_T^{0.5}(R_{ij})=\frac{1}{6}\left(I_{ij1}+I_{ij2}\right)+\frac{1}{4}\left(J_{ij1}+J_{ij2}\right)+\frac{1}{2}\left(Q_{ij}+Z_{ij}\right) \tag{10}$$

Second, the membership functions of $S_i=\sum_{j=1}^{n}\left(d_{ij}\otimes w_j\right)$ can also be developed through arithmetic operations of fuzzy numbers [23] as $S_i=\left[\sum_{j=1}^{n}(b_{ij}-a_{ij})(e_j-d_j)\alpha^2+\sum_{j=1}^{n}\left(a_{ij}(e_j-d_j)+d_j(b_{ij}-a_{ij})\right)\alpha+\sum_{j=1}^{n}a_{ij}d_j,\right.$ $\left.\sum_{j=1}^{n}(b_{ij}-c_{ij})(e_j-f_j)\alpha^2+\sum_{j=1}^{n}\left(c_{ij}(e_j-f_j)+f_j(b_{ij}-c_{ij})\right)\alpha+\sum_{j=1}^{n}c_{ij}f_j\right]$.

Assume that $I_{i1}=\sum_{j=1}^{n}(\mathrm{b}_{ij}-\mathrm{a}_{ij})(e_j-d_j)$, $J_{i1}=\sum_{j=1}^{n}\left(\mathrm{a}_{ij}(e_j-d_j)+d_j(\mathrm{b}_{ij}-\mathrm{a}_{ij})\right)$, $I_{i2}=\sum_{j=1}^{n}(\mathrm{b}_{ij}-\mathrm{c}_{ij})(e_j-f_j)$, $J_{i2}=\sum_{j=1}^{n}\left(\mathrm{c}_{ij}(e_j-f_j)+f_j(\mathrm{b}_{ij}-\mathrm{c}_{ij})\right)$, $Q_i=\sum_{j=1}^{n}\mathrm{a}_{ij}d_j$, $Y_i=\sum_{j=1}^{n}\mathrm{b}_{ij}.e_j$, $Z_i=\sum_{j=1}^{n}\mathrm{c}_{ij}f_j$. The left and right membership functions of S_i can be produced as follows.

$$f_{S_i}^{L}(x)=\frac{-J_{i1}+\left[J_{i1}^{2}-4I_{i1}\left(Q_i-x\right)\right]^{1/2}}{2I_{i1}},\ Q_i\le x\le Y_i \tag{11}$$

$$f_{S_i}^{R}(x)=\frac{-J_{i2}-\left[J_{i2}^{2}-4I_{i2}\left(Z_i-x\right)\right]^{1/2}}{2I_{i2}},\ Y_i\le x\le Z_i \tag{12}$$

$S_i=\left(S_i^a,S_i^b,S_i^c\right)=[Q_i,Y_i,Z_i]$. According to [7], the left inverse function of can obtained as $g_{S_i}^{L}(y)=I_{i1}y^2+J_{i1}y+Q_i$ and the right inverse function of can obtained as $g_{S_i}^{R}(y)=I_{i2}y^2+J_{i2}y+Z_i$. The left integral value can be produced as $I_L(S_i)=\frac{1}{3}I_{i1}+\frac{1}{2}J_{i1}+Q_i$ and the right integral value can be produced as $I_R(S_i)=\frac{1}{3}I_{i2}+\frac{1}{2}J_{i2}+\mathrm{Z}_i$. Finally, the total integral value can be derived as equation (13). The defuzzification value of $S_i=\left(S_i^a,S_i^b,S_i^c\right)$, denoted as $\bar{S}_i$, can then be obtained by $I_T^{0.5}(S_i)$.

$$I_T^{0.5}(S_i)=\frac{1}{6}\left(I_{i1}+I_{i2}\right)+\frac{1}{4}\left(J_{i1}+J_{i2}\right)+\frac{1}{2}\left(Q_i+\mathrm{Z}_i\right). \tag{13}$$

Step 7. Compute the value of $\bar{Q}_i$.

$$\bar{Q}_i=\frac{\upsilon\left(\bar{S}_i-\bar{S}^{*}\right)}{\left(\bar{S}^{\circ}-\bar{S}^{*}\right)}\oplus\frac{(1-\upsilon)\left(\bar{R}_i^{\circ}-\bar{R}_i^{\circ *}\right)}{\left(\bar{R}_i^{\circ}-\bar{R}_i^{*}\right)} \tag{14}$$

where $\bar{S}^{*}=\min_i \bar{S}_i$, $\bar{S}^{\circ}=\max_i \bar{S}_i$, $\bar{R}_i^{\circ}=\max_j \bar{R}_{ij}$, $\bar{R}_i^{*}=\min_j \bar{R}_{ij}$, $\bar{R}_i^{\circ *}=\min_i \bar{R}_i^{\circ}$ and herein $\upsilon=(n+1)/2n$. If $\bar{Q}_i$ is smaller, the alternative A_i, $i=1\sim m$, has higher ranking order.

4. Numerical Example

Assume that a manufacturing firm is considering to purchase an advanced, high-quality demand forecasting software to predict the future market share of its products. Further assume that a committee of three decision-makers, D_1, D_2, D_3, of this firm is formed to conduct the evaluation of five different softwares of five companies, A_i, $i=1\sim 5$, using this proposed method. The committee has reached a consensus to use three qualitative criteria, including reputation (C_1), user-friendly (C_2), service (C_3), and two quantitative criteria, including number of functions (C_4, unit: fuzzy #) and cost (C_5, unit: fuzzy \$10,000), to conduct the evaluation. Moreover, suppose that the ratings of alternatives versus qualitative criteria are assessed in linguistic values represented using triangular fuzzy numbers, including VP (very poor), P (poor), M (moderate), G (good), VG = (very good); and VP = (0.1,0.2,0.3), P = (0.2,0.35,0.5), M = (0.35,0.55,0.75), G = (0.6,0.75,0.9) and VG = (0.8,0.9,1); in addition, the “importance” is a linguistic variable whose values include

UI (unimportant), LI (less important), IM (important), MI (more important) and VI (very important). These linguistic values are further represented by triangular fuzzy numbers, including UI = (0.125,0.25,0.35), LI = (0.25,0.4,0.55), I = (0.4,0.575,0.725), MI = (0.575,0.725,0.875) and VI = (0.775,0.875,1).

Suppose the linguistic values of the three quantitative criteria (C_1, C_2, C_3) of the committee are shown in Table 1 and the averaged ratings can be obtained by (1) as also shown in Table 1. The values of quantitative criteria (C_4, C_5) are shown in Table 2 and the normalized values can be obtained by Eqs. (2) and (3) as also shown in Table 2. By Eqs. (4) and (5), $f_j^* = (a_j^*, b_j^*, c_j^*)$ and $f_j^\circ = (a_j^\circ, b_j^\circ, c_j^\circ)$ can be obtained as shown in Table 3. The fuzzy difference d_{ij} can be obtained by (6) as shown in Table 4. The fuzzy weights given are shown in Table 5 and the averaged fuzzy weights can be obtained by Eq. (7) as also shown in Table 5.

By Eqs. (8)–(10), the left integral value $I_L(R_{ij})$, the right integral value $I_R(R_{ij})$ and the total integral value $I_T^{0.5}(R_{ij})$ can be produced as shown in Tables 6–8, respectively. The $\bar{R}_i^\circ$ and $\bar{R}_i^*$ can be obtained from Table 8 as shown in Table 9

By Eqs. (11)–(13), the left integral value $I_L(S_i)$, the right integral value $I_R(S_i)$ and the total integral value $I_T^{0.5}(S_i)$ can be produced as shown in Table 10. By Table 10, $\bar{S}^\circ = 3.5710$ and $\bar{S}^* = 2.1272$. By Eq. (14), $\bar{Q}_i$ can be produced as shown in Table 11. According to Table 11, the ranking order of the five alternatives is $A_2 > A_1 > A_3 > A_4 > A_5$.

Table 1. Ratings and averaged ratings

A.	***C.***	***D1***	***D2***	***D3***	**Averaged ratings**
A_1	C_1	G	VG	G	(0.6667,0.8000,0.9333)
	C_2	VG	VG	G	(0.7333,0.8500,0.9667)
	C_3	G	VG	M	(0.5833,0.7333,0.8833)
A_2	C_1	VG	VG	G	(0.7333,0.8500,0.9667)
	C_2	G	VG	VG	(0.7333,0.8500,0.9667)
	C_3	M	G	G	(0.5157,0.6833,0.8500)
A_3	C_1	G	G	G	(0.6000,0.7500,0.9000)
	C_2	G	VG	VG	(0.7333,0.8500,0.9667)
	C_3	G	VG	G	(0.6667,0.8000,0.9333)
A_4	C_1	M	G	G	(0.5167,0.6833,0.8500)
	C_2	VG	G	G	(0.6667,0.8000,0.9333)
	C_3	M	M	P	(0.3000,0.4833,0.6667)
A_5	C_1	M	G	M	(0.4333,0.6167,0.8000)
	C_2	P	M	P	(0.2500,0.4167,0.5833)
	C_3	P	M	M	(0.3000,0.4833,0.6667)

Table 2. Quantitative values/Normalized values

A.	***C4***	***C5***	***C4***	***C5***
A_1	(18,20,21)	(1.5,1.6,1.8)	(0.3000,0.5000,0.6000)	(0.4545,0.6364,0.7273)
A_2	(20,22,25)	(1.6,1.8,2.1)	(0.5000,0.7000,1.0000)	(0.1818,0.4545,0.6364)
A_3	(17,18,22)	(1.5,1.7,1.9)	(0.6667,0.8000,0.9333)	(0.3636,0.5455,0.7273)
A_4	(16,17,20)	(1.8,2.1,2.3)	(0.1000,0.2000,0.5000)	(0.0000,0.1818,0.4545)
A_5	(15,16,17)	(1.2,1.5,1.7)	(0.0000,0.1000,0.2000)	(0.5455,0.7273,1.0000)

Table 3. The fuzzy best value and fuzzy worst value

C.	$f_j^* = (a_j^*, b_j^*, c_j^*)$	$f_j^\circ = (a_j^\circ, b_j^\circ, c_j^\circ)$
C_1	(0.7333,0.8500,0.9667)	(0.4333,0.6167,0.8000)
C_2	(0.7333,0.8500,0.9667)	(0.2500,0.4167,0.5833)
C_3	(0.6667,0.8000,0.9333)	(0.3000,0.4833,0.6667)
C_4	(0.5000,0.7000,1.000)	(0.0000,0.1000,0.2000)
C_5	(0.5455,0.7273,1.0000)	(0.0000,0.1818,0.4545)

Table 4. The fuzzy difference

C.	A_1	A_2	A_3	A_4	A_5
C_1	(0.1250,0.5938,1.0625)	(0.0625,0.5000,0.9375)	(0.1875,0.6875,1.1875)	(0.2813,0.8125,1.3438)	(0.3750,0.9375,1.5000)
C_2	(0.1744,0.5000,0.8256)	(0.1744,0.5000,0.8256)	(0.1744,0.5000,0.8256)	(0.2209,0.5698,0.9186)	(0.7093,1.1047,1.5000)
C_3	(0.1579,0.6053,1.0526)	(0.2105,0.6842,1.1579)	(0.0789,0.5000,0.9211)	(0.5000,1.0000,1.5000)	(0.5000,1.0000,1.5000)
C_4	(0.4000,0.7000,0.9000)	(0.0000,0.5000,1.0000)	(0.3000,0.9000,1.3000)	(0.5000,1.0000,1.4000)	(0.8000,1.1000,1.5000)
C_5	(0.3182,0.5909,0.7727)	(0.4091,0.7727,1.3182)	(0.3182,0.6818,1.1364)	(0.5909,1.0455,1.5000)	(0.0455,0.5000,0.9545)

Table 5. Importance weights and average weights

C.	D_1	D_2	D_3	**Average Weights**
C_1	MI	IM	IM	(0.4583,0.6250,0.7750)
C_2	VI	VI	VI	(0.7750,0.8750,1.0000)
C_3	MI	VI	MI	(0.6417,0.7750,0.9167)
C_4	VI	MI	VI	(0.7083,0.8250,0.9583)
C_5	LI	IM	UI	(0.2583,0.4083,0.5417)

Table 6. The left integral value

$I_L(R_{ij})$	A_1	A_2	A_3	A_4	A_5
C_2	0.2012	0.1584	0.2439	0.3036	0.3633
C_2	0.2809	0.2809	0.2809	0.3291	0.7516
C_3	0.2753	0.3221	0.2097	0.5368	0.5368
C_4	0.4246	0.1965	0.4658	0.5799	0.7313
C_5	0.1549	0.2015	0.1712	0.2784	0.0966

Table 7. The right integral value

$I_R(R_{ij})$	A_1	A_2	A_3	A_4	A_5
C_1	0.5855	0.50859	0.6625	0.76133	0.8602
C_2	0.6248	0.62476	0.62476	0.70131	1.225
C_3	0.7064	0.78465	0.60596	1.06319	1.0632
C_4	0.7156	0.67431	0.98528	1.07444	1.1636
C_5	0.3259	0.50265	0.43687	0.6096	0.3505

Table 8. The total integral value

$I_T^{0.5}(R_{ij})$	A_1	A_2	A_3	A_4	A_5
C_1	0.3934	0.3335	0.4532	0.5325	0.6117
C_2	0.4528	0.4528	0.4528	0.5152	0.9883
C_3	0.4908	0.5534	0.4078	0.8000	0.8000
C_4	0.5701	0.4354	0.7256	0.8272	0.9474
C_5	0.2404	0.3521	0.3040	0.4440	0.2235

Table 9. $\bar{R}_i^{\circ}$ and $\bar{R}_i^{*}$

	A_1	A_2	A_3	A_4	A_5
$\bar{R}_i^{\circ}$	0.5701	0.5534	0.7256	0.8272	0.9883
$\bar{R}_i^{*}$	0.2404	0.3335	0.3040	0.4440	0.2235

Table 10. $I_L(S_i)$, $I_R(S_i)$ and $I_T^{0.5}(S_i)$

	A_1	A_2	A_3	A_4	A_5
$I_L(S_i)$	1.3368	1.1595	1.3716	2.0277	2.4795
$I_R(S_i)$	2.9582	3.09496	3.31536	4.20987	4.6625
$I_T^{0.5}(S_i)$	2.1475	2.1272	2.3435	3.1188	3.5710

Table 11. $\bar{Q}_i$

$\bar{Q}_1$	$\bar{Q}_2$	$\bar{Q}_3$	$\bar{Q}_4$	$\bar{Q}_5$
0.0287	0.0000	0.2532	0.6979	0.8275

5. Conclusion

A fuzzy VIKOR extension was proposed in this paper, in which ratings of alternatives versus qualitative criteria and criteria weights were assessed in linguistic values represented using positive triangular fuzzy numbers. Values under quantitative criteria were also assessed by positive triangular fuzzy numbers. Membership functions of fuzzy weighted ratings were derived by the α-cuts of a fuzzy number and arithmetic operations of fuzzy numbers for better performing ranking procedure. The fuzzy difference d_{ij} was added by a smallest value in the fuzzy difference matrix, i.e., $-\delta$, to avoid negative values to better executing multiplication of two fuzzy numbers. Meanwhile, the total integral value was used to defuzzify and rank fuzzy numbers to complete the model.

The normalization method, arithmetic operations of fuzzy numbers through α-cuts, total integral value, etc. used in this paper were different from that used in existing fuzzy VIKOR methods; comparison with existing methods cannot display convincing advantage of the correct ranking result obtained by the proposed method; thus, a comparison was not conducted. Nevertheless, the developed equations, such as Eqs. (8), (9), (11) and (12), have shown that the proposed extension solves the limitation of using approximation to obtain the multiplication of fuzzy ratings and fuzzy weights in some fuzzy VIKOR methods. In addition, a numerical example has been conducted to demonstrate the feasibility of the proposed fuzzy VIKOR extension. However, the ranking result may differ if the fuzzy ratings, fuzzy weights, normalization method and ranking method, etc. differ. In future research, a software can be developed to help decision-makers perform the decision-making process better.

Conflict of Interest

The author declares no conflict of interest.

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References

[1] Opricovic, S., & Tzeng, G. H. (2004). Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research, 156(2),* 445–455.

[2] Shemshadi, A., Shirazi, H., Toreihi, M., & Tarokh, M. (2011). A fuzzy VIKOR method for supplier selection based on entropy measure for objective weighting. *Expert Systems with Applications, 38(10),* 12160–12167.

[3] Ploskas, N., & Papathanasiou, J. (2019). A decision support system for multiple criteria alternative ranking using TOPSIS and VIKOR in fuzzy and nonfuzzy environments. *Fuzzy Sets and Systems, 377*, 1–30.

[4] Opricovic, S. (2011). Fuzzy VIKOR with an application to water resources planning. *Expert Systems with Applications, 38(10),* 12983–12990.

[5] Taghavifard, M. T., & Majidian, S. (2022). Identifying cloud computing risks based on firm's ambidexterity performance using fuzzy VIKOR technique. *Global Journal* of *Flexible Systems Management*, *23*,113–133.

[6] Devi, K. (2011). Extension of VIKOR method in intuitionistic fuzzy environment for robot selection. *Expert Systems with Applications, 38(11),* 14163–14168.

[7] Liou, T. S., & Wang, M. J. (1992). Ranking fuzzy numbers with integral value. *Fuzzy Sets and Systems, 50(3),* 247–255.

[8] Duckstein, L., & Opricovic, S. (1980). Multi-objective optimization in river basin development. *Water Resources Research*, *16(1),* 14–20.

[9] Opricovic, S. (1998). *Multicriteria Optimization of Civil Engineering Systems*. Ph.D. Thesis, University of Belgrade, Faculty of Civil Engineering, Belgrade, Serbia.

[10] Opricovic, S., & Tzeng, G. H. (2007). Extended VIKOR method in comparison with outranking methods. *European Journal of Operational Research, 178(2),* 514–529.

[11] Rathore, R., Thakkar, J. J., & Jha, J. K. (2019). Evaluation of risks in foodgrains supply chain using failure mode effect analysis and fuzzy VIKOR. *International Journal of Quality & Reliability Management*, *38(2),* 551–580.

[12] Le, H. T., & Chu, T. C. (2023). Ranking alternatives using a fuzzy preference relation-based fuzzy VIKOR method. *Axioms*, *12(10),* 1–18.

[13] Delaram, J., Valilai, O. F., Houshamand, M., & Ashtiani, F. (2021). A matching mechanism for public cloud manufacturing platforms using intuitionistic fuzzy VIKOR and deferred acceptance algorithm. *International Journal of Management Science and Engineering Management, 16(2),* 107–122.

[14] Rahmi, B. A. K. İ. (2022). Extended VIKOR method based on interval-valued intuitionistic fuzzy numbers for selection of logistics centre location. *Journal of the Human and Social Science Researches*, *11(3),* 1821–1837.

[15] Salimian, S., Mousavi, S. M., & Antucheviciene, J. (2022). An interval-valued intuitionistic fuzzy model based on extended VIKOR and MARCOS for sustainable supplier selection in organ transplantation networks for healthcare devices. *Sustainability*, *14(7),* 1–21.

[16] Gupta, P., Mehlawat, M. K., & Ahemad, F. (2023). Selection of renewable energy sources: A novel VIKOR approach in an intuitionistic fuzzy linguistic environment. *Environment, Development and Sustainability*, *25,* 3429–3467.

[17] Dagistanli, H. A. (2024). An interval-valued intuitionistic fuzzy VIKOR approach for R&D project selection in defense industry investment decisions. *Journal of Soft Computing and Decision Analytics*, *2(1),* 1–13.

[18] Chu, T. C., & Le, T. H. P. (2022). Evaluating and selecting agricultural insurance packages through an AHP-based fuzzy TOPSIS method. *Soft Computing*, *26(15)*, 7339–7354.

[19] Ma, Z. M., Yang, W., Li, L., & Xu, Z. (2023). Ranking fuzzy numbers using additive priority degrees. *Expert Systems with Applications*, *234,* 121019.

[20] Bihari, R., Jeevaraj, S., & Kumar, A. (2023). Geometric approach for ranking generalized trapezoidal fuzzy numbers and its application in selecting security guard service company. *Expert Systems with Applications*, *234,* 121052.

[21] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8(3),* 338–353.

[22] Dubois, D., & Prade, H. (1978). Operations on fuzzy numbers. *International Journal of Systems Science*, *9(6)*, 613–626.

[23] Kaufmann, A., & Gupta, M. (1991). *Introduction to Fuzzy Arithmetic Theory and Applications*. New York: Van Nostrand Reinhold.

[24] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning, part 1, 2 and 3. *Information Science*, *8(3),* 199–249; *8(4),* 301–357; *9(1),* 43–80.

[25] Chu, T. C., & Charnsethikul, P. (2013). Ordering alternatives under fuzzy multiple criteria decision making via a fuzzy number dominance based ranking approach. *International Journal of Fuzzy Systems, 15(3),* 263–273.