Modified Laplace Decomposition Method for Lane-Emden Type Differential Equations

Fu-Kang Yin, Wang-Yi Han, and Jun-Qiang Song

Abstract—In this paper, A new scheme, deduced from the Modified Laplace decomposition method (MLDM), is presented to obtain exact solutions for the singular initial value problems (IVPs) of lane-Emden type. Both linear and nonlinear cases are considered. If the exact solution exists in the zeroth component, the proposed method yields exact solution in two iterations. The results show that MLDM is very effective and easy to implement.

Index Terms—Modified laplace decomposition method, singular, initial value problem, lane-emden type equation.

I. INTRODUCTION

Lane-Emden type initial value problems (IVPs) have found a wide range of applications in modeling a class of problems in the field of mathematical physics and astrophysics [1], [2], which can be written in the form:

$$y'' + \frac{2}{x}y' + f(y) = 0, \quad 0 \le x \le 1 \tag{1}$$

subject to conditions

$$y(0) = A, \quad y'(0) = B \tag{2}$$

where A and B are constants. $f(y)$ is a real-valued continuous function. This equation is very useful in the study of various models such as the theory of stellar structure, the thermal behavior of a spherical cloud of gas, isothermal gas spheres and the theory of thermionic currents [1]-[3].

On the other hand, attentions have been paid to another class of singular initial value problems of Lane-Emden type with the form:

$$y'' + \frac{2}{x}y' + f(x,y) = g(x), \quad 0 \le x \le 1 \tag{3}$$

The solution of the Lane-Emden problem is numerically challenging because of the singularity behavior at the origin. The approximate solutions of the Lane-Emden equation were given by Adomian decomposition method [4]-[6], homotopy perturbation method [7]-[9], variational iteration method

[10]-[12], differential transformation method [13], Wavelets-Collocation method [14], [15] and so on.

Laplace Adomian decomposition method (LADM) was first proposed by Suheil A. Khuri [16], [17] and has been successfully used to find the solution of differential equations [18]-[23]. The major advantage of this method is its capability of combining the two powerful methods to obtain exact solutions for nonlinear equations. However, LADM will generate "noise term" [24] for inhomogeneous equations. Therefore, M. Hussain [25] developed a modified Laplace decomposition method (MLDM) which can accelerate the rapid convergence of series solution when compared with Laplace Adomian decomposition method. In this paper, we will apply the MLDM to obtain exact or approximate analytical solutions of the Lane-Emden type equations.

II. MODIFIED LAPLACE DECOMPOSITION METHOD

In this section, we will briefly discuss the use of the MLDM for the solution of Lane-Emden equation given in (3).

Multiplying x and then taking the Laplace transform on both sides of (3) gives:

$$-s^2 L'\{y\} - y(0) + L\{xf(x,y) - xg(x)\} = 0, \tag{4}$$

where L is the operator of Laplace transform and $L'\{y\} = \dfrac{dL\{y\}}{ds}$.

We decompose $f(x,y)$ into two parts:

$$f(x,y) = R[y(x)] + N[y(x)], \tag{5}$$

where $R[y(x)]$ and $N[y(x)]$ denote the linear term and the nonlinear term respectively.

The Adomian decomposition method and the Adomian polynomials can be used to handle Eq. (4) and to address the nonlinear term $N[y(x)]$. MLDM defines a solution $y(x)$ by an infinite series of components given by:

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \tag{6}$$

and the nonlinear term can be represented by an infinite series of the Adomian polynomials A_n in the form:

$$N\{y(x)\} = \sum_{n=0}^{\infty} A_n(x), \tag{7}$$

where A_n are the Adomian polynomials and it can be calculated by formula given below:

$$A_n = \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} N\left(\sum_{n=0}^{\infty} \lambda^n u_n \right) \right]_{\lambda=0} , \quad n = 0,1,2,\cdots. \quad (8)$$

Therefore Adomian's polynomials are given by:

$$
\begin{aligned}
A_0 &= N[u_0], \\
A_1 &= u_1 N'[u_0], \\
A_2 &= u_2 N'[u_0] + \frac{1}{2!} u_1^2 N''[u_0], \\
A_3 &= u_3 N'[u_0] + u_1 u_2 N''[u_0] + \frac{1}{3!} u_1^3 N'''[u_0], \\
&\vdots
\end{aligned}
\quad (9)
$$

After substituting (6) and (7) into (4), we have

$$
\left.
\begin{aligned}
&-s^2 L'\left\{ \sum_{n=0}^{\infty} y_n(x) \right\} - y(0) - L\{xg(x)\} \\
&+ L\left\{ xR\left[\sum_{n=0}^{\infty} y_n(x) \right] + x \sum_{n=0}^{\infty} A_n(x) \right\}
\end{aligned}
\right\} = 0. \quad (10)
$$

Using the linearity of Laplace transform, it follows that

$$
\left.
\begin{aligned}
&-s^2 \sum_{n=0}^{\infty} L'\{y_n(x)\} - y(0) - L\{xg(x)\} \\
&+ \sum_{n=0}^{\infty} L\{xR[y_n(x)] + xA_n(x)\}
\end{aligned}
\right\} = 0, \quad (11)
$$

In general, the recursive relation is given by:

$$
\begin{aligned}
L'\{y_0(x)\} &= -s^{-2} y(0) - s^{-2} L\{xg(x)\}, \\
L'\{y_{n+1}(x)\} &= s^{-2} L\{xR[y_n(x)] + xA_n(x)\},
\end{aligned}
\quad (12)
$$

By integrating both sides of Eq. (12) from 0 to s respect with s, we have

$$
\begin{aligned}
L\{y_0(x)\} &= \int \left[-s^{-2} y(0) - s^{-2} L\{xg(x)\} \right] ds, \\
L\{y_{n+1}(x)\} &= \int s^{-2} L\{xR[y_n(x)] + xA_n(x)\} ds,
\end{aligned}
\quad (13)
$$

Taking the inverse Laplace transform to Eq. (13), one obtains

$$
\begin{aligned}
y_0(x) &= L^{-1}\left\{ \int \left[-s^{-2} y(0) - s^{-2} L\{xg(x)\} \right] ds \right\} = H(x), \\
y_{n+1}(x) &= L^{-1}\left\{ \int s^{-2} L\{xR[y_n(x)] + xA_n(x)\} ds \right\},
\end{aligned}
\quad (14)
$$

where $H(x)$ represents the term arising from source term and prescribed initial condition. The initial solution is important, and the choice of Eq. (14) as the initial solution always leads to noise oscillation during the iteration procedure.

In order overcome the shortcoming, we assume that $H(x)$ can be divided into the sum of two parts namely $H_0(x)$ and $H_1(x)$, therefore we get

$$H(x) = H_0(x) + H_1(x). \quad (15)$$

Instead of the iteration procedure Eq. (14), we suggest the following modification

$$
\begin{aligned}
y_0(x) &= H_0(x), \\
y_1(x) &= H_1(x) + L^{-1}\left\{ \int s^{-2} L\{xR[y_n(x)] + xA_n(x)\} ds \right\}, \quad (16) \\
y_{n+1}(x) &= L^{-1}\left\{ \int s^{-2} L\{xR[y_n(x)] + xA_n(x)\} ds \right\},
\end{aligned}
$$

The solution through the modified Laplace decomposition method highly depends upon the choice of $H_0(x)$ and $H_1(x)$.

III. ILLUSTRATIVE EXAMPLES

In this section, we will apply the method presented in this paper to solve singular IVPs of Lane-Emden-type.

A. Linear Cases

Example 1. Consider the following linear, homogeneous Lane-Emden differential equation (5, 9, 11, 26, 27, 29)

$$y'' + \frac{2}{t} y' - 2(2t^2 + 3) y = 0, \quad (17)$$

with initial conditions

$$y(0) = 1, \quad y'(0) = 0. \quad (18)$$

According to the MLDM and initial conditions, we get

$$-s^2 L'\{y\} - 1 = L\{2(2t^3 + 3t) y\}, \quad (19)$$

and then, the recursive relations can be obtained as

$$
\begin{aligned}
y_0(t) &= L^{-1}\left\{ \int \left[-s^{-2} y(0) \right] ds \right\}, \\
y_n(t) &= L^{-1}\left\{ \int -s^{-2} L\{2(2t^3 + 3t) y_{n-1}(t)\} ds \right\},
\end{aligned}
\quad (20)
$$

Considering equation (20), starting with substituting the value of $y(0) = 1$ given in (18) into (20), we have

$$
\begin{aligned}
y_0 &= 1, \\
y_1 &= t^2 + \frac{1}{5} t^4, \\
y_2 &= \frac{3}{10} t^4 + \frac{13}{105} t^6 + \frac{1}{90} t^8 \quad (21) \\
y_3 &= \frac{3}{70} t^6 + \frac{17}{630} t^8 + \frac{59}{11550} t^{10} + \frac{1}{3510} t^{12}, \\
&\vdots
\end{aligned}
$$

Hence, the solution series in general gives

$$y = y_1 + y_2 + y_3 + \cdots = 1 + t^2 + \frac{1}{2} t^4 + \frac{1}{6} t^6 \cdots, \quad (22)$$

The closed form of the series (22) is $y(t) = \exp(t^2)$ which gives an exact solution of the problem.

It should be noted that the MLDM gives an analytical solution in the closed form like the Adomian decomposition method [5], homotopy perturbation method [9] and variational iteration method [11].

Example 2. Consider the linear, non-homogenous Lane-Emden differential equation (14)

$$y'' + \frac{2}{t} y' + y^n = 0, \quad t \geq 0, \quad n = 0,1 \qquad (23)$$

subject to initial conditions

$$y(0) = 1, \quad y'(0) = 0. \qquad (24)$$

According to the MLDM and initial conditions (24), we have

$$-s^2 L'\{y\} - y(0) + L\{ty^n\} = 0. \qquad (25)$$

when $n = 0$, we obtain

$$-s^2 L'\{y\} - y(0) + L\{t\} = 0, \qquad (26)$$

We east get

$$L'\{y\} = -\frac{1}{s^2} + \frac{1}{s^4}, \quad L\{y\} = \frac{1}{s} - \frac{1}{3s^3}, \quad y(t) = 1 - \frac{t^2}{6}, \quad (27)$$

which the exact solution.

When $n = 1$, we obtain

$$-s^2 L'\{y\} - 1 + L\{ty\} = 0, \qquad (28)$$

the recursive relation is obtained as

$$y_0(t) = L^{-1}\left\{\int \left[-s^{-2} y(0)\right] ds\right\}, \\ y_n(t) = L^{-1}\left\{\int -s^{-2} L\{ty_{n-1}(t)\} ds\right\}, \qquad (29)$$

By using Eq. (29), we can get

$$y_0 = 1, \quad y_1 = -\frac{t^2}{3 \times 2!}, \quad y_2 = \frac{t^4}{5 \times 4!}, \quad y_3 = -\frac{t^6}{7 \times 6!}, \cdots \quad (30)$$

Hence, the solution series in general gives

$$y = 1 - \frac{t^2}{3 \times 2!} + \frac{t^2}{5 \times 4!} - \frac{t^6}{7 \times 6!} + \cdots, \qquad (31)$$

The closed form of the series (31) is $y(t) = \sin t / t$ which gives an exact solution of the problem.

Example 3. Consider the linear, non-homogenous Lane-Emden equation (5, 11, 26, 27):

$$y'' + \frac{2}{t} y' + y = 6 + 12t + t^2 + t^3, \quad 0 < t \leq 2 \qquad (32)$$

subject to initial conditions

$$y(0) = 0, \quad y'(0) = 0. \qquad (33)$$

According to the MLDM and initial conditions (33), we have

$$-s^2 L'\{y\} + L\{ty\} - L\{6t + 12t^2 + t^3 + t^4\} = 0, \quad (34)$$

the recursive relation is obtained as

$$y_0(t) = L^{-1}\left\{\int -s^{-2} L\{6t + 12t^2\} ds\right\}, \\ y_1(t) = L^{-1}\left\{\int s^{-2} \{L\{ty_0\} - L\{t^3 + t^4\}\} ds\right\}, \qquad (35) \\ y_n(t) = L^{-1}\left\{\int s^{-2} L\{ty_{n-1}\} ds\right\},$$

And then we have

$$y_0 = t^2 + t^3, \, y_1 = 0, \, y_n = 0, \qquad (36)$$

so the exact solution is obtained as $y = t^2 + t^3$.

Unlike the variational iteration method [11] which introduces noise terms, our method can get the exact solution while only need a few iterations.

B. Nonlinear Cases

Example 4. Consider the following nonlinear, homogeneous Lane-Emden differential equation

$$y'' + \frac{2}{t} y' + y^3 - (6 + t^6) = 0, \quad t \geq 0 \qquad (37)$$

subject to initial conditions

$$y(0) = 0, \quad y'(0) = 0. \qquad (38)$$

According to the MLDM and initial conditions (38), we have

$$-s^2 L'\{y\} + L\{ty^3\} - L\{6t + t^7\} = 0, \qquad (39)$$

the recursive relation is obtained as

$$L'\{y_0\} = -s^{-2} L\{6t\}, \\ L'\{y_1\} = s^{-2} L\{ty_{n-1}^3\} - L\{t^7\}, \qquad (40) \\ L'\{y_n\} = s^{-2} L\{ty_{n-1}^3\},$$

so we have

$$L\{y_0\} = \frac{2}{s^3}, \quad y_0 = t^2, \\ L\{y_1\} = 0, \quad y_1 = 0, \qquad (41) \\ L\{y_n\} = 0, \quad y_n = 0, \quad n > 1$$

so the exact solution is obtained as $y = t^2$.

Example 5. Consider the following nonlinear,

homogeneous Lane-Emden differential equation (9, 11, [27]-[29]:

$$y'' + \frac{2}{t} y' + 4\left(2e^y + e^{y/2}\right) = 0, \quad 0 \le t \le 1, \tag{42}$$

subject to initial conditions

$$y(0) = 0, \quad y'(0) = 0. \tag{43}$$

The exact solution is $y(t) = -2\ln\left(1 + t^2\right)$.

Applying the MLDM and initial conditions (43), we have

$$-s^2 L'\{y\} + L\left\{4t\left(2e^y + e^{y/2}\right)\right\} = 0, \tag{44}$$

and then, we get the recursive relation as

$$L'\{y_0\} = 0, \quad L'\{y_n\} = s^{-2} L\{4tA_{n-1}\}, \tag{45}$$

where the nonlinear operator $N[y] = 2e^y + e^{y/2}$ is decomposed as in (9) in terms of the Adomian polynomials. From (10) the first few Adomian polynomials for $N[y] = 2e^y + e^{y/2}$ are computed as follows:

$$A_0 = 2e^{y_0} + e^{y_0/2},$$
$$A_1 = y_1\left(2e^{y_0} + \frac{1}{2}e^{y_0/2}\right),$$
$$A_2 = y_2\left(2e^{y_0} + \frac{1}{2}e^{y_0/2}\right) + \frac{y_1^2}{2!}\left(2e^{y_0} + \frac{1}{4}e^{y_0/2}\right),$$
$$A_3 = y_3\left(2e^{y_0} + \frac{1}{2}e^{y_0/2}\right) + y_1 y_2\left(2e^{y_0} + \frac{1}{4}e^{y_0/2}\right)$$
$$\quad + \frac{y_1^3}{3!}\left(2e^{y_0} + \frac{1}{8}e^{y_0/2}\right), \tag{46}$$
$$\vdots$$

And then by using (45), we have

$$L'\{y_0\} = 0, \qquad L\{y_0\} = 0, \qquad y_0 = 0,$$
$$L'\{y_1\} = \frac{12 \times 1!}{s^4}, \quad L\{y_1\} = -\frac{12 \times 1!}{3s^3}, \quad y_1 = -2t^2,$$
$$L'\{y_2\} = -\frac{20 \times 3!}{s^6}, L\{y_2\} = -\frac{20 \times 3!}{5s^5}, y_2 = t^4,$$
$$L'\{y_3\} = \frac{28 \times 5!}{s^8}, L\{y_3\} = -\frac{28 \times 5!}{7s^7}, y_3 = -\frac{2}{3}t^6, \tag{47}$$
$$L'\{y_4\} = -\frac{36 \times 7!}{s^{10}}, L\{y_4\} = \frac{36 \times 7!}{9s^9}, \quad y_3 = \frac{1}{2}t^8,$$

Then the solution in a series form is given by

$$y(t) = -2\left(t^2 - \frac{1}{2}t^4 + \frac{1}{3}t^6 - \frac{1}{4}t^8 + \cdots\right). \tag{48}$$

Hence the exact solution has the form

$$y(t) = -2\ln\left(1 + t^2\right). \tag{49}$$

Example 6. Consider the nonlinear, homogeneous Lane-Emden differential equation (9, 11, 27)

$$y'' + \frac{2}{t} y' - 6y = 4y \ln y, \quad 0 \le t \le 1, \tag{50}$$

subject to initial conditions

$$y(0) = 1, \quad y'(0) = 0. \tag{51}$$

The exact solution is $y(t) = e^{t^2}$.

Applying the MLDM and initial conditions (51), we have

$$-s^2 L'\{y\} - 1 = L\{6yt + 4yt \ln y\}, \tag{52}$$

and then, we get the recursive relation as

$$L'\{y_0\} = -s^{-2}, \quad L'\{y_n\} = s^{-2} L\{6y_{n-1}t + 4y_{n-1}t \ln y\}, \tag{53}$$

where the nonlinear operator $N[y] = y \ln y$ is decomposed as in (9) in terms of the Adomian polynomials. From (10) the first few Adomian polynomials for $N[y] = y \ln y$ are computed as follows:

$$A_0 = y_0 \ln y_0,$$
$$A_1 = y_1\left(1 + \ln y_0\right),$$
$$A_2 = y_2\left(1 + \ln y_0\right) + \frac{y_1^2}{2y_0}, \tag{54}$$
$$A_3 = y_3\left(1 + \ln y_0\right) + \frac{y_1 y_2}{y_0} - \frac{y_1^3}{6y_0^2},$$
$$\vdots$$

According to Eq. (53) and by using the Eq. (54), we can get the series solution as

$$L'\{y_0\} = -\frac{1}{s^2}, \qquad L\{y_0\} = \frac{1}{s}, \qquad y_0 = 1,$$
$$L'\{y_1\} = -\frac{6 \times 1!}{s^4}, \quad L\{y_1\} = \frac{6 \times 1!}{3s^3}, \quad y_1 = t^2,$$
$$L'\{y_2\} = -\frac{10 \times 3!}{s^6}, \quad L\{y_2\} = \frac{10 \times 3!}{5s^5}, y_2 = \frac{1}{2!}t^4, \tag{55}$$
$$L'\{y_3\} = -\frac{7 \times 5!}{s^8}, \qquad L\{y_3\} = \frac{7 \times 5!}{7s^7}, \quad y_3 = \frac{1}{3!}t^6,$$
$$L'\{y_4\} = -\frac{3 \times 7!}{s^{10}}, \quad L\{y_3\} = \frac{3 \times 7!}{9s^9}, \quad y_3 = \frac{1}{4!}t^8,$$

which is the expansion of the function $y(t) = \exp\left(t^2\right)$ and is the exact solution of example 6.

The series solutions obtained by the MLDM are the same as Adomian decomposition method [28], variational iteration method [11] and homotopy perturbation method [9]. Unlike other approximation methods [14], [15], [27], [28], our method can get the exact solution.

IV. CONCLUSION

In this paper, we have successfully employed the Modified Laplace decomposition method (MLDM) to obtain exact solutions for singular IVPs of Lane-Emden-type. MLDM can accelerate the rapid convergence of series solution when compared with Laplace decomposition method. It is shown that the MLDM is a promising tool for singular IVP's of Lane-Emden type, and in some cases, yields exact solutions in two iterations.

ACKNOWLEDGMENT

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