Model Identification and Feedforward Control for Non-minimum Phase Systems

Hashimah Ismail, Norlela Ishak, Mazidah Tajjudin, Mohd Hezri Fazalul Rahiman, and Ramli Adnan

Abstract—High precision control has received a great attention. Nowadays, as the machines are used for precise operations in manufacturing field, fast and precision control is required. This paper presents model identification using Matlab System Identification Toolbox from open loop input-output experimental data of a linear servo system. Two different models are obtained by varying the sampling time of the experiments by 40ms and 50ms. Different sampling time changes the position of non-stable zero of the transfer function. The identified models are tested using pole-placement feedback controller. The system is further improved by introducing feedforward Zero Phase Error Tracking Control (ZPETC) to control the position tracking of non-minimum phase system. The variations of system performances for both systems are analyzed through simulation and real-time experiment. The results verify that conventional ZPETC is more effective for system that has non-stable zero far from the unit circle.

Index Terms—Feedforward control, non-minimum phase, pole-placement, system identification, Zero Phase Error Tracking Control (ZPETC).

I. INTRODUCTION

Basically, mathematical model of a physical systems are obtained using Physic law and differential equation. Many researchers use this common technique. One example of this method in deriving an ac servo motor is from Huang and Chen [1]. He finally described the discrete time model of the system as:

\[ Y(k) = -a_1 Y(k-1) - a_2 Y(k-2) + b_1 u(k-d-1) + b_2 u(k-d-2) \]

(1)

The mathematical model also can be derived using System identification technique which a model is constructed from observed input and output data.

The construction of model from data involves a data set, a set of candidate model and a rule by which candidate models can be assessed like Least Square selection rule. With the availability of the modern tools such as Matlab System Identification Toolbox and LabVIEW System Identification Toolkit, model identification using experimental data is easy to be implemented. Currently, many researchers keen on using Matlab System Identification Toolbox to approximate their plant models as in [2]-[5].

In this paper, two non-minimum phase systems are considered. Both systems are obtained by changing the sampling time during open loop data collection of an ac servomotor. Using 50ms sampling time, the non-stable zero is far from unity and as the sampling time is reduced to 40ms, the zero moves near to the unit circle. Thus, feedforward controller for the discrete linear non-minimum phase systems (NMP) is developed. However, NMP zeros are difficult to be compensated perfectly by classical feedforward control because the direct inverse of zeros become unstable poles [6]. Fortunately, zero phase error tracking control provides a strategy to control NMP system by eliminating phase error caused by NMP zeros. This control technique is proposed by Tomizuka in 1987 [6]. This strategy is developed and implemented to the systems. The performances of both systems are analyzed and compared. Feedforward control needs a feedback control system to compensate the error and in this study, pole-placement method is used.

This paper is organized by the following: Section II describes the system plant and hardware setup; Section III describes System Identification technique that is used to find the model transfer function; Section IV describes the feedback and feedforward controller design; Section V is on results and discussion and Section VI is the conclusion.

II. PLANT AND HARDWARE SETUP

The plant used in this research is X-Y table a.c servo system. Basic hardware for x-y table consists of two servomotors, two servo drivers, two linear encoders, two linear precision sensors, motion card, data acquisition card and a host PC set. The linear sensor and encoder resolution is 1 µm. The minimum movement is 1µm. In this paper, only x-axis is considered. For system control and interfacing, Microsoft Visual C++ 6.0 is used. Matlab software is used for model identification and results analysis. The whole picture of system plant is given in Fig. 1.

III. SYSTEM IDENTIFICATION

The plant model is derived using Matlab System Identification from the measured input and output signals of a real plant shown in Section II. Complete procedure on this method is obtained from [7]. The data collection for the input-output open loop test of the plant was done using Visual C++ console programming and data acquisition were done through Advantech PCI-1716 interface card and Advantech PCI2140 motion card. The input signal was generated using three different frequencies that based on equation (2).
\[ u(k) = \sum_{i=1}^{c} a_i \cos \omega_i t_i \]  \hspace{1cm} (2)

where, \( a_i \): amplitude; \( \omega_i \): frequency; \( t_i \): sampling time (sec);

The value of \( p \) in equation one is determined by the number of model parameter that needs to be identified. In this study, as three different frequencies were used for input signal, the model that can be obtained is limited to second and third order only \([2]\). The voltage input signal used for this process is shown in Fig.2 below.

Using the voltage input in Fig.1, the linear displacements of the x-axis were measured. The experiments were running two times by varying the sampling time. The displacement results for 40ms and 50ms sampling time are shown in Fig 3.

Using this technique, the input and output signals of Fig. 2 and Fig. 3 have to be divided into two parts, i.e. (1—500) samples and (501—1000) samples. The first part of the input-output signals will be used to obtain the plant model and the second part of the input-output signals will be used to validate the obtained model. Using Matlab System Identification Toolbox, the first part of the input-output signal produces a plant model. In this study, the third-order Auto-Regressive with Exogenous Input (ARX) 331 parametric model was selected to represent the nearest model of true plant. The ARX parametric model is chosen because this structure is represented by a simple linear differential equation. Noise is assumed to be null. Higher-order models may produce unstable output. S.Banks et.al \([8]\) stated that the high order model makes application of advanced control techniques inefficient and often results in high order controllers which can not meet the real-time requirement. The identified plant models for 40ms and 50 ms sampling time using system identification technique are as in equation 3 and 4.

\[ \frac{B_c(z^{-1})}{A_c(z^{-1})} = \frac{0.8548z^{-1} + 1.874z^{-2} - 0.7743z^{-3}}{1 - 0.6781z^{-1} - 0.3582z^{-2} + 0.03685z^{-3}} \]  \hspace{1cm} (3)

\[ \frac{B_c(z^{-1})}{A_c(z^{-1})} = \frac{1.524z^{-1} + 1.292z^{-2} - 0.4552z^{-3}}{1 - 0.7218z^{-1} - 0.3163z^{-2} + 0.03812z^{-3}} \]  \hspace{1cm} (4)

The second part of input-output signals will be used to validate the obtained model of equation 3 and equation 4. The second part of the input signal is used as an input to the model and the output from the model will be compared with the second part of the output signal. The result can be seen in Fig. 4 for 40ms and Fig. 5 for 50ms sampling time. The best fits are 95.2% and 94.83% respectively which are very much acceptable.

Using model selection criterions, the following information as in Table I were obtained.

<table>
<thead>
<tr>
<th>Sampling time</th>
<th>40ms</th>
<th>50ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss Function</td>
<td>0.18523</td>
<td>0.184117</td>
</tr>
<tr>
<td>Akaike’s Final Prediction Error, FPE</td>
<td>0.189676</td>
<td>0.188853</td>
</tr>
</tbody>
</table>

Based on the small values of Akaike’s Final Prediction (FPE) and Best Fit percentage value, the models are considered acceptable. A detail on Akaike’s Final Prediction Error (FPE) is in \([9]\).

The plant models transfer function poles and zeroes plots are shown in Fig. 6. It is shown that both systems are non-minimum phase system because they have a zero outside the unit circle. The zero location of the system may influence the tracking performance.
IV. CONTROLLER DESIGN

This section presents the development and implementation of the controller design. Feedback and feedforward controller are applied to the system. The general block diagram of the control system is given in Fig. 7.

A. Feedback Controller

Pole-placement method is used for feedback control. This method enables all poles of the closed-loop to be placed at desired location to produce stable output performance. The detail of feedback controller block diagram is presented in Fig. 8.

\[
\frac{Y(z^{-1})}{U(z^{-1})} = \frac{Kz^{-1}B(z^{-1})}{A(z^{-1})F(z^{-1}) + B(z^{-1})G(z^{-1})}
\]

(5)

where:

\[
A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + \cdots + a_nz^{-n}
\]

\[
B(z^{-1}) = b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + \cdots + b_nz^{-n}
\]

\[
F(z^{-1}) = 1 + f_1z^{-1} + f_2z^{-2} + f_3z^{-3} + \cdots + f_nz^{-n-1}
\]

\[
G(z^{-1}) = g_0 + g_1z^{-1} + g_2z^{-2} + g_3z^{-3} + \cdots + g_nz^{-n-1}
\]

Using Diophantine equation to solve for \(F(z^{-1})\) and \(G(z^{-1})\), and \(T(z^{-1})\) is the location of poles that were required. Using \(T(z^{-1})=1+tz^{-1}\), only one pole position is considered at \(t_i=-p\) which is inside the unity circle. Other poles cancelled each others. The range of \(p\) is \(0<p<1\). For slow response, \(p\) is set large and for fast response, \(p\) is set small. The forward gain is given as \(K_f=\text{Sum}(T)/\text{Sum}(B_T)\). From Eq. (6) the following matrix equation can be derived:

\[
A(z^{-1})F(z^{-1}) + B(z^{-1})G(z^{-1}) = T(z^{-1})
\]

(6)

Let \(EM\) where \(E\) is a Sylvester Matrix given by:

\[
E = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_2 & a_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_3 & a_2 & a_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_4 & a_3 & a_2 & a_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_5 & a_4 & a_3 & a_2 & a_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & a_n & a_{n-1} & a_{n-2} & a_{n-3} & a_{n-4} & \cdots & a_1 & 1 & 0 & 0 & 0 \\
0 & 0 & a_n & a_{n-1} & a_{n-2} & a_{n-3} & \cdots & a_2 & a_1 & 1 & 0 & 0 \\
0 & 0 & 0 & a_n & a_{n-1} & a_{n-2} & \cdots & a_3 & a_2 & a_1 & 1 & 0 \\
0 & 0 & 0 & 0 & a_n & a_{n-1} & \cdots & a_4 & a_3 & a_2 & a_1 & 1 \\
\end{bmatrix}
\]

(8)

\[
M = \begin{bmatrix}
f_1 \\
f_2 \\
f_3 \\
f_4 \\
\vdots \\
f_{m+1} \\
g_0 \\
g_1 \\
g_2 \\
g_3 \\
g_4 \\
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
\text{I} - a_1 \\
-a_2 \\
-a_3 \\
-a_4 \\
\vdots \\
-a_n \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

(9)
Thus, vector $F$ and $G$ can be computed from $M=E^{-1}D$. The computed parameters for the respective models are as the following:

Model 40ms:

$$T=1-0.755z^{-1}$$
$$K_f = 0.1254$$
$$F(z^{-1}) = -0.1888 - 0.01084 z^{-1}$$
$$G(z^{-1}) = 0.1309 - 0.05082 z^{-1} - 0.0005088 z^{-2} \quad (10)$$

Model 50ms:

$$T=1-0.84z^{-1}$$
$$K_f = 0.0678$$
$$F(z^{-1}) = -0.2803 - 0.04053z^{-1}$$
$$G(z^{-1}) = 0.1064 - 0.042 z^{-1} - 0.003392 z^{-2} \quad (11)$$

B. Feedforward Controller

For non-minimum phase systems, feedforward controller is suitable to improve the system performance. In most ideal situation, combination of feedback and feedforward control can entirely eliminate the effect of measured disturbance on the process output. If following a sophisticated command input desired, a feedback controller alone is often no longer sufficient and a command feedforward controller is needed [10]. There are many types of feedforward controller with their own advantages and limitations. This paper only focuses on implementing feedforward Zero Phase Error Tracking Control (ZPETC).

To implement the feedforward ZPETC, let the closed-loop transfer function of the system without feedback controller is given by:

$$G_c(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})}$$

$$A_c = 1 + a_1z^{-1} + a_2z^{-2} + \ldots + a_nz^{-n}$$

$$B_c(z^{-1}) = b_0 + b_1z^{-1} + b_2z^{-2} + \ldots + b_nz^{-n}$$

$$d \quad \text{= time delay}$$

The function $B_c(z^{-1})$ can be factorized into minimum and non-minimum phase factors by:

$$B_c(z^{-1}) = B_m(z^{-1}) B_n(z^{-1}) \quad (13)$$

$B_m(z^{-1})$ denotes the minimum phase factor and $B_n(z^{-1})$ denotes the non-minimum phase factor. The structure of ZPETC is illustrated by the following block diagram:

\[ \text{Fig. 9. ZPETC structure block diagram.} \]

V. RESULT AND DISCUSSION

Initially, both identified models have been tested by simulation with pole-placement feedback control. This is to find the best value of $p$. Using $p = 0.755$ for 40ms model and $p = 0.84$ for 50ms model, the controller produced good performance. The input used in the testing are square wave form. The results are as shown in Fig. 10 below.

Then, both model is tested with feedback and feedforward control by simulation and real-time experiment. The input desired as in equation 14 is used in the testing. The input equation has multiple frequency and amplitude. The simulation and real-time experimental results using ZPETC controller are presented in Fig. 11 and Fig. 12. From the figures, it is observed that, for step time of 200 – 500 the tracking error is getting bigger when the tracking frequency is larger. This is due to the controller could not provide unity gain at high frequency.

$$r(k) = 5 + 20 \sin (0.01 \cos (0.05 (t + 80))) \quad (14)$$

The root mean squared error (RMSE) for the all testings are summarized in Table II.

\[ \text{Fig. 10. Pole-placement feedback control performance.} \]

\[ \text{Fig. 11. ZPETC control performance for 40ms sampling time.} \]

\[ \text{Fig. 12. ZPETC control performance for 50ms sampling time.} \]

\[ \text{TABLE II: RMSE SUMMARY} \]

<table>
<thead>
<tr>
<th>RMSE</th>
<th>NMP systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation</td>
<td>0.08230</td>
</tr>
<tr>
<td>Real-Time Experiment</td>
<td>0.1677</td>
</tr>
</tbody>
</table>

Based on the RMSE in Table II, it is shown that by using small sampling time, the position tracking performance is better. Small sampling time 40ms shows better rmse about 50% improvements for simulation and 40% improvement for real-time experiment. This is because the zero will be positioned far away from the unit circle which will increase
the rate of system decay and system is said to be more stable.

Theoretically, the systems performance can be described using frequency response gain plot. From Fig. 13, unity gain can be achieved at very slow frequency for both models but as the frequency goes higher, gain drop for 50ms sampling time is bigger compared to 40ms sampling time model. This shows that as the non-minimum phase or non-stable zero goes far from the unit circle, the system is said to be more unstable and the gain will be dropped.

VI. CONCLUSION

Plant modeling using Matlab System Identification Toolbox from open loop input-output experimental data is successfully obtained and presented. This method is simpler as compared to deriving mathematical model using physic laws. The models are also successfully tested using pole-placement feedback controller and the results show an outstanding performances. Conventional ZPETC Tomizuka(1987) controller is developed and applied to the systems by simulation and real-time application. The performances for both systems using multiple frequency sinusoidal input are analyzed and compared. The performances of simulation and experiment show that rmse for 40ms sampling time system is less than 50ms sampling time. This is because of rate of system decay is faster for system that has non-stable zero far from the unit circle. It also proves that as the non-minimum phase or non-stable zero goes far from the unit circle, the system is said to be more unstable and the gain for high frequency will be dropped.

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REFERENCES


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